Glossary of Z notation

Names

\[ a, b \] identifiers
\[ d, e \] declarations (e.g., \( a : A; b, \ldots : B \ldots \))
\[ f, g \] functions
\[ m, n \] numbers
\[ p, q \] predicates
\[ s, t \] sequences
\[ x, y, \ldots \] expressions
\[ A, B \] sets
\[ C, D \] bags
\[ Q, R \] relations
\[ S, T \] schemas
\[ X \] schema text (e.g., \( d, d \mid p \) or \( S \))

Definitions

\[ a := x \] Abbreviation definition
\[ a ::= b \mid \ldots \] Free type definition (or \( a ::= b \| x \mid \ldots \))
\[ [a] \] Introduction of a given set (or \( [a, \ldots ] \))
\[ -a \] Prefix operator
\[ _a \] Postfix operator
\[ _a^\text{p} \] Infix operator

Logic

\[ true \] Logical true constant
\[ false \] Logical false constant
\[ \neg p \] Logical negation
\[ p \land q \] Logical conjunction
\[ p \lor q \] Logical disjunction
\[ p \Rightarrow q \] Logical implication (\( \neg p \lor q \))
\[ p \Leftrightarrow q \] Logical equivalence (\( p \Rightarrow q \land q \Rightarrow p \))
\[ \forall X \cdot q \] Universal quantification
\[ \exists X \cdot q \] Existential quantification
\[ \exists_1 X \cdot q \] Unique existential quantification
\[ \text{let } a := x; \ldots \bullet p \] Local definition

Sets and expressions

\[ x = y \] Equality of expressions
\[ x \neq y \] Inequality (\( \neg (x = y) \))
\[ x \in A \] Set membership
\[ x \notin A \] Non-membership (\( \neg (x \in A) \))
\[ \emptyset \] Empty set
\[ A \subseteq B \] Set inclusion
\[ A \subset B \] Strict set inclusion (\( A \subseteq B \land A \neq B \))
\[ \{ x, y, \ldots \} \] Set of elements
\[ \{ X \bullet x \} \] Set comprehension
\[ \lambda X \cdot x \] Lambda-expression – function
\[ \mu X \cdot x \] Mu-expression – unique value

let \( a := x; \ldots \bullet y \) Local definition
if \( p \) then \( x \) else \( y \) Conditional expression
\( (x, y, \ldots ) \) Ordered tuple
\( A \times B \times \ldots \) Cartesian product
\( P \mid A \) Power set (set of subsets)
\( P_1 \mid A \) Non-empty power set
\( F \mid A \) Set of finite subsets
\( F_1 \mid A \) Non-empty set of finite subsets
\( A \setminus B \) Set intersection
\( A \cup B \) Set union
\( A \setminus B \) Set difference
\( \bigcup A \) Generalized union of a set of sets
\( \bigcap A \) Generalized intersection of a set of sets
\( \text{first } x \) First element of an ordered pair
\( \text{second } x \) Second element of an ordered pair
\( \#A \) Size of a finite set

Relations

\( A \leftrightarrow B \) Relation (\( P \mid A \times B \))
\( a \mapsto b \) Maplet (\( (a, b) \))
\( \text{dom } R \) Domain of a relation
\( \text{ran } R \) Range of a relation
\( \text{id } A \) Identity relation
\( Q \upharpoonright R \) Forward relational composition
\( Q \circ R \) Backward relational composition (\( R \upharpoonright Q \))
\( A \restriction R \) Domain restriction
\( A \vartriangleleft R \) Domain anti-restriction
\( R \triangleright A \) Range restriction
\( R \triangleright A \) Range anti-restriction
\( R \mid A \) Relational image
\( \text{iter}_n R \) Relation composed \( n \) times
\( R^n \) Same as \( \text{iter}_n R \)
\( R^\sim \) Inverse of relation (\( R^{-1} \))
\( R^* \) Reflexive-transitive closure
\( R^+ \) Irreflexive-transitive closure
\( Q \oplus R \) Relational overriding (\( (\text{dom } R \vartriangleleft Q) \cup R \))
\( a \text{ R } b \) Infix relation

Functions

\( A \rightarrow B \) Partial functions
\( A \rightarrow B \) Total functions
\( A \rightarrow B \) Partial injections
\( A \rightarrow B \) Total injections
\( A \rightarrow B \) Partial surjections
\( A \rightarrow B \) Total surjections
\( A \rightarrow B \) Bijective functions
\( A \rightarrow B \) Finite partial functions
\( A \rightarrow B \) Finite partial injections
\( f x \) Function application (or \( f(x) \))
Numbers

\( \mathbb{Z} \) Set of integers

\( \mathbb{N} \) Set of natural numbers \( \{0, 1, 2, \ldots\} \)

\( \mathbb{N}_1 \) Set of non-zero natural numbers \( (\mathbb{N} \setminus \{0\}) \)

\( m + n \) Addition

\( m - n \) Subtraction

\( m \times n \) Multiplication

\( m \div n \) Division

\( m \mod n \) Modulo arithmetic

\( m \leq n \) Less than or equal

\( m < n \) Less than

\( m \geq n \) Greater than or equal

\( m > n \) Greater than

\( \text{succ } n \) Successor function \( \{0 \mapsto 1, 1 \mapsto 2, \ldots\} \)

\( m \ldots n \) Number range

\( \text{min } A \) Minimum of a set of numbers

\( \text{max } A \) Maximum of a set of numbers

Sequences

seq \( A \) Set of finite sequences

seql \( A \) Set of non-empty finite sequences

iseq \( A \) Set of finite injective sequences

\( \emptyset \) Empty sequence

\( \langle x, y, \ldots \rangle \) Sequence \( \{1 \mapsto x, 2 \mapsto y, \ldots\} \)

\( s \upharpoonright t \) Sequence concatenation

\( \sim / s \) Distributed sequence concatenation

head \( s \) First element of sequence \( (s(1)) \)

tail \( s \) All but the head element of a sequence

\( \text{last } s \) Last element of sequence \( (s(#s)) \)

\( \text{front } s \) All but the last element of a sequence

\( \text{rev } s \) Reverse a sequence

\( \text{squash } f \) Compact a function to a sequence

\( A \mid s \) Sequence extraction \( (\text{squash}(A \downarrow s)) \)

\( s \mid A \) Sequence filtering \( (\text{squash}(s \triangleright A)) \)

\( s \text{ prefix } t \) Sequence prefix relation \( (s \uparrow v = t) \)

\( s \text{ suffix } t \) Sequence suffix relation \( (u \uparrow s = t) \)

\( s \text{ in } t \) Sequence segment relation \( (u \uparrow s \uparrow v = t) \)

disjoint \( A \) Disjointness of an indexed family of sets

\( A \text{ partition } B \) Partition an indexed family of sets

Bags

bag \( A \) Set of bags or multisets \( (A \mapsto \mathbb{N}_1) \)

\( \emptyset \) Empty bag

\( \langle x, y, \ldots \rangle \) Bag \( \{x \mapsto 1, y \mapsto 1, \ldots\} \)

count \( C \times x \) Multiplicity of an element in a bag

\( C \times x \) Same as count \( C \times x \)

\( n \odot C \) Bag scaling of multiplicity

\( x \in C \) Bag membership

\( C \sqcap D \) Sub-bag relation

\( C \sqcup D \) Bag union

\( C \sqsubseteq D \) Bag difference

\text{items } s \) Bag of elements in a sequence

Schema notation

**Vertical schema.**

\[
\begin{align*}
S &\quad \text{New lines denote } \\
\frac{d}{p} &\quad \text{and '}
\wedge\text{'. The schema name and predicate part are optional. The schema}
\text{may subsequently be referenced by name in the document.}
\end{align*}
\]

**Axiomatic definition.**

\[
\begin{align*}
\frac{d}{p} &\quad \text{The definitions may be non-unique. The predicate}
\text{part is optional. The definitions apply globally in the document.}
\end{align*}
\]

\[
\begin{align*}
[\alpha, \ldots] &\quad \text{The generic parameters are optional. The definitions}
\text{must be unique. The definitions apply globally in the document.}
\end{align*}
\]

\( S \equiv [X] \) Horizontal schema

\( [T; \ldots; \ldots] \) Schema inclusion

\( \zeta . a \) Component selection (given \( z : S \))

\( \theta S \) Tuple of components

\( \neg S \) Schema negation

\( \text{pre } S \) Schema precondition

\( S \land T \) Schema conjunction

\( S \lor T \) Schema disjunction

\( S \Rightarrow T \) Schema implication

\( S \iff T \) Schema equivalence

\( S \setminus (\alpha, \ldots) \) Hiding of component(s)

\( S \mid T \) Projection of components

\( S \circ T \) Schema composition \( (S \text{ then } T) \)

\( S \gg T \) Schema piping \( (S \text{ outputs to } T \text{ inputs})\)

\( S[a/b, \ldots] \) Schema component renaming \((b \text{ becomes } a, \\text{etc.})\)

\( \forall X \bullet S \) Schema universal quantification

\( \exists X \bullet S \) Schema existential quantification

\( \exists_1 X \bullet S \) Schema unique existential quantification

**Conventions**

\( a? \) Input to an operation

\( a! \) Output from an operation

\( a \) State component before an operation

\( a' \) State component after an operation

\( S \) State schema before an operation

\( S' \) State schema after an operation

\( \Delta S \) Change of state (normally \( S \land S' \))

\( \Xi S \) No change of state (normally \( [S \land S'][\theta S = \theta S'] \))

**URL:** [http://www.cs.reading.ac.uk/people/jpb/](http://www.cs.reading.ac.uk/people/jpb/)